Pakistan Journal of Social Research
ISSN 2710-3129 (P) 2710-3137 (O)
Vol. 5, No. 2, June 2023, pp. 1337-1346.
https://doi.org/10.52567/pjsr.v5i02.1359
www.pjsr.com.pk

HYBRID DOUBLE EXPONENTIALLY WEIGHTED MOVING AVERAGE (HDEWMA) CONTROL CHART FOR INVERSE RAYLEIGH DISTRIBUTION

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ABSTRACT
In this paper, we proposed a Hybrid Double Exponentially Weighted Moving Average HDEWMA control chart. The proposed control chart is based on Inverse Rayleigh Distributed lifetimes using simple random sampling (SRS) and ranked set sampling (RSS). Out-of-control-Average Run Length (ARL_1) is used to evaluate the performance of the proposed control chart. The HDEWMA control chart is compared with traditional/simple EWMA and CUSUM control charts. The performance of the control chart is evaluated using out of control average run length (ARL_1). A simulated example is used to compare the proposed HDEWMA, traditional/simple EWMA chart and CUSUM control chart. It is observed that the proposed HDEWMA control chart performs simple EWMA and CUSUM control charts. The HDEWMA control chart can be used for efficient monitoring of the production process in manufacturing industries where the data is coming from inverse Rayleigh Distribution.
Keywords: Hybrid, EWMA, CUSUM, DEWMA, average run length (ARL).

1. INTRODUCTION

The most effectual method in SPC for perpetuate process is control charting. The idea of control charts was introduced by Shewhart (1931) proposed control chart has an assumption that the data should follow the Normal distribution. The 99.73% of observations that are sampled from a Normal distribution are included within 3 standard deviations of the mean, i.e. between the limits $\mu \pm 3\sigma$. Observations are said to be in-control if they remain within these control limits and the process goes out of control when the observations plots outside these limits.

The use of statistical quality control charts in many sectors of life revealed that nearly all of the control charts are meant to offer data on the quality characteristic/studied variable. If we can obtain some information about another variable(s) that is connected with our variable of interest, we may use the efficient charting statistic to improve the control chart's efficiency. The additional information is known as auxiliary information, and the variable that provides it is known as auxiliary variable. This additional/auxiliary information is utilized to estimate the unknown parameters at various times in the survey sampling process (Montgomery (2009a).

Haq (2013) suggested the New Hybrid-EWMA (HEWMA) chart for the process mean. The control chart's performance is monitored using the Average Run Length (ARL). The Average Run Length is the average number of points drawn on a control chart before an out of control condition is signaled. Montgomery (2009) considers the control chart with the lowest ARL_1 score to be one of the best options for detecting assignable cause. Rayleigh distribution, one of the well-known continuous distributions created by Lord Rayleigh, was discussed by Hussain and Ahmad (2014). Raza et al. (2015) suggestted EWMA and DEWMA control charts for gamma-distributed lifetimes with Type-I censoring.

The concept of conditional anticipated values was employed to keep track of the mean level. The average run length was used as a metric in the performance assessments. Raza et al. (2016) wanted to look at EWMA control charts with Type-I censoring for Poisson-exponential lifetimes. Simple random sampling and rank set sampling, two extensively used sample strategies, were utilised. Monitoring mean level shifts with censored data piqued the interest of numerous applicable challenges.

Alkahtani (2013) investigated the effect of non-normality on the average run length (ARL) performance of EWMA and DEWMA using various skewed (Gamma) and symmetric non-normal (t) distributions. The paper dealt with the modelling of heterogeneity in lifetime processes using the mixture of the inverse Rayleigh distribution, and the focus was on the Bayesian inference of the mixture model using non-informative (Jeffreys and uniform) and informative (gamma) priors. Ali (2015), Abdelkhalek (2020), Alevizakos and Koukouvinos (2020) and Shahid and Rahee (2019) proposed and studied a new flexible extension of the inverse Rayleigh model.

Abbas et al. (2017) explained process monitoring is an essential element for an improved quality of final products. A variety of tools were used for it; control charts were one of these choices.

Raza et al. (2018), Abbasi et al. (2018) stated the process structures of manufacturing industry efficiently modeled control charts. Kowsalya et al. (2019) developed a new attribute control chart based on inverse Rayleigh distribution under a time truncated life test. Feng et al. (2020) proposed a test statistic based on the "divide-and-conquer" strategy, and integrated this into the multivariate EWMA charting scheme for Phase II process monitoring.

The main ideology was to calculate and combine the T_2 statistics on low-dimensional sub-vectors together. Essentially the proposed procedure was distribution-free and computation efficient. Raza et al. (2020) proposed HDEWMA chart to monitor the mean of Weibull distribution in the presence of type-I censored data. Their focus was to use the conditional median (CM) for the imputation of censored observations. The performance of the control chart was determined by the average run length (ARL). They compared CM-DEWMA control chart with CM-based HDEWMA control chart.

In this paper we propose a Hybrid Double Exponentially Weighted Moving Average HDEWMA control chart by inverse Rayleigh distribution. The performance of the proposed chart will be determined using out-of-control-Average Run Length (ARL_1).

2. METHODOLOGY

2.1 The Proposed Hybrid Double EWMA Control Chart

For monitoring the data generated from Inverse Rayleigh Distribution we define a hybrid DEWMA control chart. The PDF of the inverse Rayleigh distribution with scale parameter θ is:

$$f(x) = \left(\frac{2\theta}{x^3}\right) \exp\left(-\frac{\theta}{x^2}\right) \qquad x, \theta > 0 \qquad (2.1.1)$$

The corresponding cumulative distribution function is,

$$F(x) = \exp\left(-\frac{\theta}{x^2}\right) \qquad x, \theta > 0 \qquad (2.1.2)$$

Let $X_1, X_2, ...,$ be a sequence of IID random variables from the Inverse Rayleigh Distribution. For Smoothing Parameters $\lambda_1, \lambda_2, \lambda_3 \in [0,1]$, we define two new sequences $E_1, E_2, ...,$ and $HE_1, HE_2, ...,$ as follow:

$$E_t = \lambda_1 \overline{X}_i + (1-\lambda_1) E_{t-1} \qquad (2.1.3)$$

Now, the plotting statistic of DEWMA is calculated as:

$$HE_t = \lambda_2 E_t + (1-\lambda_2) HE_{t-1} \qquad (2.1.4)$$

Now, we define control charting statistics for Hybrid DEWMA, that is as follow:

$$HDE_t = \lambda_3 HE_t + (1-\lambda_3) HDE_{t-1} \qquad (2.1.5)$$

Compute the HDEWMA statistics for different subgroup size m and sample size n. Take the $(1-p)-\text{th}$ quantile point of the charting statistics values to define the UCL of the chart. Where, p is specified as 0.0027. (Montgomery et al. 1994).

The control limits of the proposed chart will be derived as:

$$E(HDE_t) = \mu$$

And variance of HE_t(cf. Haq, 2017).

$$V(HDE_t) = \frac{\lambda_1^2 \lambda_2^2 \; MSE}{(\lambda_1 - \lambda_2)^2} \left[\frac{(1-\lambda_1)^2 (1-(1-\lambda_1)^{2i})}{1-(1-\lambda_1)^2} + \frac{(1-\lambda_2)^2 (1-(1-\lambda_2)^{2i})}{1-(1-\lambda_2)^2} - \frac{2(1-\lambda_1)(1-\lambda_2)\left\{1-(1-\lambda_1)^i (1-\lambda_2)^i\right\}}{1-(1-\lambda_1)(1-\lambda_2)} \right]$$

So control limits are:

UCL= $\mu_0 + L_1 \sqrt{Var(HDE_t)}$

LCL= $\mu_0 - L_2 \sqrt{Var(HDE_t)}$

Where L_1 and L_2 shows the 99.73 quantile point of the distribution under study.

Algorithm 1:

1. Initiate the algorithm by fixing the sample size, n and group size m. Estimate the parameter using phase-I data if it is unknown.
2. The sample is selected using SRS and RSS schemes.
3. Calculate the charting statistics of proposed HDEWMA statistics for given sample size "n". Now take $(1-p)th$ quantile point, here α is predefined,
4. Repeat Step No.2 *1000 number of time*& compute the average, it will out-turn the Upper Control Limit.
5. Hatch the values of HDEWMA monitoring statistic with respect to the subgroup numbers. Record the sample number at which the monitoring statistic falls outside the control limit,
6. Repeat Step 4 a large number of times and calculate the mean of the subgroup numbers at which the process first time crossed the UCL. This will result in ARL_0 if the subgroups are generated from the in-control process. For the out-of-control ARL computation, introduce a shift in the data and test it against the control limit constructed using in-control data. Repeat steps 4-5 and calculate the average of subgroups falling outside the UCL, which is ARL_1.

3. RESULTS AND DISCUSSION

To study the features of the proposed Hybrid DEWMA control chart, we have simulated the data and applied the Hybrid DEWMA chart. The procedure proposed in section 2 is evaluated using SRS and RSS. The EWMA weight (λ_1) and HEWMA weight (λ_2) are fixed for evaluation purpose. The simulated distribution is found to follow Inverse Rayleigh distribution. Therefore, the limits are based on Inverse Rayleigh distribution. Among these, some results are presented here for discussion purposes. Table 3.1 gives the ARLs of the proposed HDEWMA control chart.

Table 3.1 ARL_1 for HDEWMA using SRS and RSS with θ=1.5, ARL_0=100.

Shift	HDEWMA (SRS)		HDEWMA (RSS)	
10%	Average	13.476	Average	10.601
	SD	23.3350	SD	22.5923
15%	Average	12.455	Average	10.140
	SD	25.3036	SD	24.4515
20%	Average	11.719	Average	9.791
	SD	25.0220	SD	22.7265

30%	Average	10.456	Average	9.044
	SD	25.3174	SD	23.2727
40%	Average	9.348	Average	8.444
	SD	26.1337	SD	22.8477

Table 3.1 represents the out of control values of ARL for HDEWMA (SRS) & HDEWMA (RSS) control charts of sample size n=3. The samples are generated from Inverse Rayleigh Distribution with parameter θ=1.5. The in-control ARL (ARL_0) is fixed at 100, We have simulated results 10,000 times using Monte Carlo Simulations (MC Simulation).

It is noticed that with a 10% increase in shift, the ARL_1 value is found to be 13.476 with SDRL is 23.335 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the ARL_1 is 10.601 with SDRL is 22.5923. With 15% increase in shift, the ARL_1 value is found to be 12.455 with SDRL=25.3036 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the ARL_1 value is 10.140 with SDRL = 24.4515. With 20% increase in shift, the ARL_1 value is found to be 11.719 with SDRL= 25.0220 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the ARL_1 is 9.791 with SDRL = 22.7265. With 30% increase in shift, the ARL_1 value is found to be 10.456 with SDRL = 25.3174 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the ARL_1 is 9.044 with SDRL = 23.2727. With 40% increase in shift, the ARL_1 value is found to be 9.348 with SDRL=26.1337 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the ARL_1 is 8.444 with SDRL = 22.8477. We can see that, for increase in shift, the ARL_1values for HDEWMA (RSS) control chart is less than HDEWMA (SRS) control chart, which shows HDEWMA using RSS performs better than the HDEWMA control chart using SRS.

Figure 3.1ARL_1 for HDEWMA using SRS and RSS with θ=1.5, ARL_0=100.

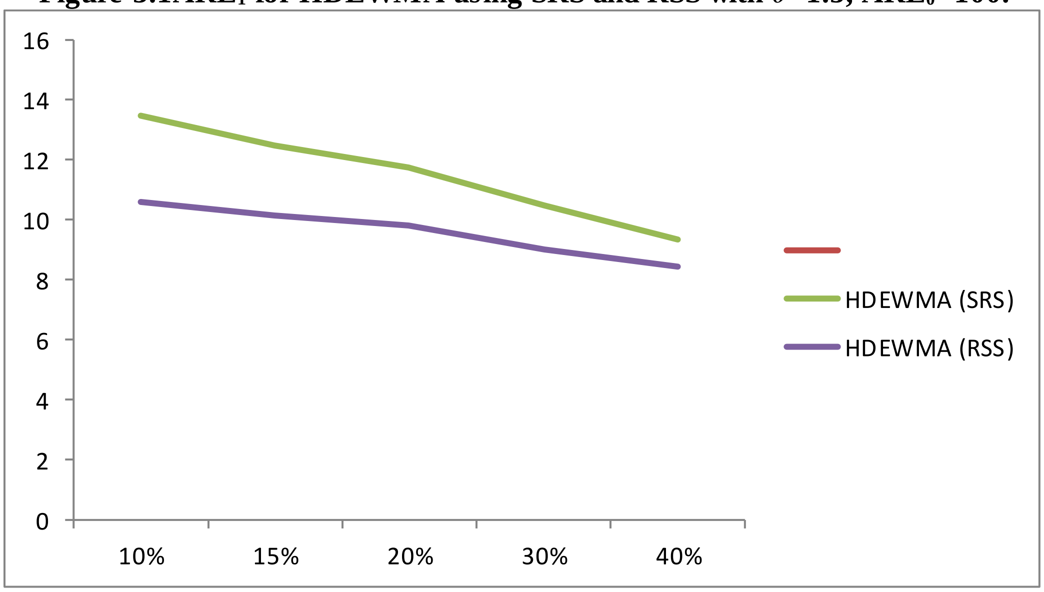

Figure 3.1 graphically represents the out-of-control values of ARL for HDEWMA (SRS) & HDEWMA (RSS) control charts of sample size n=3. The samples are generated from Inverse Reyleigh Distribution with parameter θ=1.5. The in control ARL (ARL_0) is fixed at 100, We have simulated results 10,000 times using MC Simulations.

It is noticed that with 10% increase, the ARL_1 value is found to be 13.476 with SDRL is 23.335 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the ARL_1 is 10.601 with SDRL is 22.5923. With 15% increase in shift, the ARL_1 value is found to be 12.455 with SDRL=25.3036 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the ARL_1 value is 10.140 with SDRL = 24.4515. With 20% increase in shift, the ARL_1 value is found to be 11.719 with SDRL= 25.0220 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the ARL_1 is 9.791 with SDRL = 22.7265. With 30% increase in shift, the ARL_1 value is found to be 10.456 with SDRL = 25.3174 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the ARL_1 is 9.044 with SDRL = 23.2727. With 40% increase in shift, the ARL_1 value is found to be 9.348 with SDRL=26.1337 for HDEWMA (SRS) and for HDEWMA (RSS) control chart the ARL_1 is 8.444 with SDRL = 22.8477. We can see that, for increase in shift, the ARL_1values for HDEWMA (RSS) control chart is less than HDEWMA (SRS) control chart, which shows The performance of the HDEWMA (RSS) is better than the HDEWMA (SRS).

Table 3.1.a Comparison of ARL_1 for HDEWMA, EWMA & CUSUM using SRS and RSS when θ=1.5, ARL_0=100.

Shift		HDEWMA (SRS)	HDEWMA (RSS)	EWMA (SRS)	EWMA (RSS)	CUSUM (SRS)	CUSUM (RSS)
10%	Avg	13.47	10.60	15.53	13.71	14.62	13.11
	SD	23.33	22.59	25.08	25.11	24.09	24.11
15%	Avg	12.45	10.14	14.25	13.21	13.87	12.57
	SD	25.30	24.45	24.33	24.08	23.15	23.13
20%	Avg	11.71	9.79	13.61	12.67	12.89	11.63
	SD	25.02	22.72	23.31	24.01	22.79	22.43
30%	Avg	10.45	9.04	12.50	10.81	11.66	10.23
	SD	25.31	23.27	22.22	22.76	22.26	21.79
40%	Avg	9.34	8.44	11.29	9.70	10.48	9.17
	SD	26.13	22.84	22.09	22.13	21.05	21.09

Table 3.1.a compares the out of control values of ARL for HDEWMA (SRS) and HDEWMA (RSS) with EWMA and CUSUM control charts for sample size n=3.The samples are generated from Inverse Rayleigh Distribution with parameter θ=1.5. The in control ARL (ARL_0) is fixed at 100, We have simulated results 10,000 times using MC Simulations.

We can see that, when the shift is increased, the ARL_1 values for HDEWMA control charts using rank set sampling is less than simple random sampling, which shows HDEWMA (RSS) control chart performs better than the HDEWMA (SRS) control chart. Also, it can be observed that the HDEWMA (RSS) shows minimum ARL values among the other competitor control charts i.e EWMA & CUSUM. So we can say that HDEWMA (RSS) is the most efficient control chart for the detection of shift among these. It shows better performance than EWMA and CUSUM control charts.

Table 3.2 Simulated Samples and Calculation of statistics of HDEWMA chart

Inverse Rayleigh R.No			HDEWMA Statistics	Inverse Rayleigh R.No			HDEWMA Statistics	Inverse Rayleigh R.No			HDEWMA Statistics
2.382953	0.682294	3.363658	1.833706	3.21587	2.971659	0.493318	1.233389	3.15838	1.882056	1.656595	1.289857
2.051321	2.089189	2.419881	1.213011	0.714164	3.008044	2.402361	1.493023	2.520815	3.238732	2.363489	2.260669
2.618439	1.871851	0.821133	1.213011	1.734271	0.794984	4.485471	0.21314	2.967042	1.897054	0.80298	1.91322
1.480128	0.433819	3.902656	1.807191	2.33575	2.858507	2.21381	1.492534	2.298446	0.468165	2.120194	2.189582
1.578465	2.43343	2.793356	2.425166	3.46655	0.503696	1.505475	1.953234	1.459319	0.880196	2.936515	5.107
0.548548	1.890562	2.625547	5.107	2.617836	1.656037	1.590839	5.107	1.078483	2.190758	3.635592	2.070534
0.629757	0.80534	1.995672	1.026157	2.516016	1.958422	3.559974	2.141779	2.295042	3.768096	1.977212	1.721805
1.245913	1.718476	1.833706	5.107	2.865441	1.105934	1.233389	5.107	1.693961	1.769089	1.289857	1.601136
0.412014	0.790938	1.213011	0.255681	1.357295	4.034043	1.493023	2.259583	2.579819	2.814422	2.260669	1.521777
0.726176	0.177768	2.29566	1.656595	1.556248	1.219095	0.21314	1.876204	1.659297	2.681053	1.91322	5.107

1.997 629	2.187 867	1.807 191	2.3634 89	2.128 116	1.724 023	1.492 534	5.107	2.382 953	1.763 74	2.189 582	1.1211 47
1.191 937	1.924 113	2.425 166	0.8029 8	1.286 818	2.961 882	1.953 234	1.4243 62	2.783 719	0.681 86	3.528 656	0.4933 18
3.692 983	1.586 588	4.803 78	2.1201 94	4.314 494	1.466 767	3.143 783	1.3736 74	3.702 232	4.652 499	2.070 534	2.4023 61
1.684 446	2.117 991	1.026 157	5.107	0.364 438	1.915 257	2.141 779	2.1618 03	2.398 368	1.564 775	1.721 805	5.107
1.803 541	3.191 923	2.654 436	5.107	2.961 845	0.704 21	2.696 256	1.2322 47	2.209 012	1.556 516	1.601 136	2.2138 1
2.217 185	1.364 228	0.255 681	1.9772 12	2.450 837	1.823 29	2.259 583	5.107	2.865 441	1.105 934	1.233 389	5.107
3.158 38	1.882 056	1.656 595	1.2898 57	1.819 483	1.257 233	1.876 204	1.4766 91	1.357 295	4.034 043	1.493 023	2.2595 83
2.520 815	3.238 732	2.363 489	2.2606 69	3.532 945	1.695 871	3.860 049	1.5009 75	1.556 248	1.219 095	0.213 14	1.8762 04
2.967 042	1.897 054	0.802 98	1.9132 2	2.136 282	4.096 41	1.424 362	1.4404 88	2.128 116	1.724 023	1.492 534	5.107
2.298 446	0.468 165	2.120 194	2.1895 82	1.591 475	3.597 052	1.373 674	1.2710 59	1.286 818	2.961 882	1.953 234	1.4243 62
1.459 319	0.880 196	2.936 515	5.107	1.341 987	2.994 698	2.161 803	0.8353 42	4.314 494	1.466 767	3.143 783	1.3736 74
1.078 483	2.190 758	3.635 592	2.0705 34	3.460 82	2.332 72	1.232 247	5.107	0.364 438	1.915 257	2.141 779	2.1618 03
2.295 042	3.768 096	1.977 212	1.7218 05	2.645 253	1.980 547	3.755 891	1.9465 21	2.961 845	0.704 21	2.696 256	1.2322 47
1.693 961	1.769 089	1.289 857	1.6011 36	2.057 372	3.711 336	1.476 691	5.107	2.450 837	1.823 29	2.259 583	5.107
2.579 819	2.814 422	2.260 669	1.5217 77	0.924 725	2.569 745	1.500 975	1.5126 82	1.819 483	1.257 233	1.876 204	1.4766 91
1.659 297	2.681 053	1.913 22	5.107	1.474 531	2.583 619	1.440 488	0.6735 83	3.532 945	1.695 871	3.860 049	1.5009 75
2.382 953	1.763 74	2.189 582	1.1211 47	0.634 185	1.298 928	1.271 059	5.107	2.136 282	4.096 41	1.424 362	1.4404 88
2.783 719	0.681 86	3.528 656	0.4933 18	1.054 286	2.279 96	0.835 342	1.1797 96	1.591 475	3.597 052	1.373 674	1.2710 59
3.702 232	4.652 499	2.070 534	2.4023 61	1.674 931	4.397 629	2.720 744	1.3145 67	1.341 987	2.994 698	2.161 803	0.8353 42
2.398 368	1.564 775	1.721 805	5.107	4.087 779	1.229 768	1.946 521	1.5794 2	3.460 82	2.332 72	1.232 247	5.107
2.209 012	1.556 516	1.601 136	2.2138 1	2.274 22	1.246 049	2.698 305	1.5095 25	2.645 253	1.980 547	3.755 891	1.9465 21
0.412 404	4.916 563	1.521 777	1.5054 75	3.236 14	3.220 256	1.512 682	0.9794 32	2.057 372	3.711 336	1.476 691	5.107
0.412 014	0.790 938	1.213 011	0.2556 81	1.357 295	4.034 043	1.493 023	2.2595 83	2.579 819	2.814 422	2.260 669	1.5217 77
0.726 176	0.177 768	2.295 66	1.6565 95	1.556 248	1.219 095	0.213 14	1.8762 04	1.659 297	2.681 053	1.913 22	5.107
1.997 629	2.187 867	1.807 191	2.3634 89	2.128 116	1.724 023	1.492 534	5.107	2.382 953	1.763 74	2.189 582	1.1211 47
1.191 937	1.924 113	2.425 166	0.8029 8	1.286 818	2.961 882	1.953 234	1.4243 62	2.783 719	0.681 86	3.528 656	0.4933 18

3.692	1.586	4.803	2.1201	4.314	1.466	3.143	1.3736	3.702	4.652	2.070	2.4023
983	588	78	94	494	767	783	74	232	499	534	61
1.684	2.117	1.026		0.364	1.915	2.141	2.1618	2.398	1.564	1.721	
446	991	157	5.107	438	257	779	03	368	775	805	5.107
1.803	3.191	2.654		2.961	0.704	2.696	1.2322	2.209	1.556	1.601	2.2138
541	923	436	5.107	845	21	256	47	012	516	136	1
2.217	1.364	0.255	1.9772	2.450	1.823	2.259		2.865	1.105	1.233	
185	228	681	12	837	29	583	5.107	441	934	389	5.107
3.158	1.882	1.656	1.2898	1.819	1.257	1.876	1.4766	1.357	4.034	1.493	2.2595
38	056	595	57	483	233	204	91	295	043	023	83
2.520	3.238	2.363	2.2606	3.532	1.695	3.860	1.5009	1.556	1.219	0.213	1.8762
815	732	489	69	945	871	049	75	248	095	14	04
2.967	1.897	0.802	1.9132	2.136	4.096	1.424	1.4404	2.128	1.724	1.492	
042	054	98	2	282	41	362	88	116	023	534	5.107
2.298	0.468	2.120	2.1895	1.591	3.597	1.373	1.2710	1.286	2.961	1.953	1.4243
446	165	194	82	475	052	674	59	818	882	234	62
1.459	0.880	2.936		1.341	2.994	2.161	0.8353	4.314	1.466	3.143	1.3736
319	196	515	5.107	987	698	803	42	494	767	783	74
1.078	2.190	3.635	2.0705	3.460	2.332	1.232		0.364	1.915	2.141	2.1618
483	758	592	34	82	72	247	5.107	438	257	779	03
2.295	3.768	1.977	1.7218	2.645	1.980	3.755	1.9465	2.961	0.704	2.696	1.2322
042	096	212	05	253	547	891	21	845	21	256	47
1.693	1.769	1.289	1.6011	2.057	3.711	1.476		2.450	1.823	2.259	
961	089	857	36	372	336	691	5.107	837	29	583	5.107
2.579	2.814	2.260	1.5217	0.924	2.569	1.500	1.5126	1.819	1.257	1.876	1.4766
819	422	669	77	725	745	975	82	483	233	204	91
1.659	2.681	1.913		1.474	2.583	1.440	0.6735	3.532	1.695	3.860	1.5009
297	053	22	5.107	531	619	488	83	945	871	049	75
2.382	1.763	2.189	1.1211	0.634	1.298	1.271		2.136	4.096	1.424	1.4404
953	74	582	47	185	928	059	5.107	282	41	362	88
2.783	0.681	3.528	0.4933	1.054	2.279	0.835	1.1797	1.591	3.597	1.373	1.2710
719	86	656	18	286	96	342	96	475	052	674	59
3.702	4.652	2.070	2.4023	1.674	4.397	2.720	1.3145	1.341	2.994	2.161	0.8353
232	499	534	61	931	629	744	67	987	698	803	42
2.398	1.564	1.721		4.087	1.229	1.946	1.5794	3.460	2.332	1.232	
368	775	805	5.107	779	768	521	2	82	72	247	5.107
2.209	1.556	1.601	2.2138	2.274	1.246	2.698	1.5095	2.645	1.980	3.755	1.9465
012	516	136	1	22	049	305	25	253	547	891	21
0.412	4.916	1.521	1.5054	3.236	3.220	1.512	0.9794	2.057	3.711	1.476	
404	563	777	75	14	256	682	32	372	336	691	5.107
1.097	1.222	5.066	1.5908	2.154	3.718	0.673	2.1745	0.924	2.569	1.500	1.5126
086	473	057	39	742	465	583	55	725	745	975	82
3.158	1.882	1.656	1.2898	1.819	1.257	1.876	1.4766	1.357	4.034	1.493	2.2595
38	056	595	57	483	233	204	91	295	043	023	83
2.520	3.238	2.363	2.2606	3.532	1.695	3.860	1.5009	1.556	1.219	0.213	1.8762
815	732	489	69	945	871	049	75	248	095	14	04
2.967	1.897	0.802	1.9132	2.136	4.096	1.424	1.4404	2.128	1.724	1.492	
042	054	98	2	282	41	362	88	116	023	534	5.107
2.298	0.468	2.120	2.1895	1.591	3.597	1.373	1.2710	1.286	2.961	1.953	1.4243
446	165	194	82	475	052	674	59	818	882	234	62
1.459	0.880	2.936		1.341	2.994	2.161	0.8353	4.314	1.466	3.143	1.3736
319	196	515	5.107	987	698	803	42	494	767	783	74

1.078	2.190	3.635	2.0705	3.460	2.332	1.232		0.364	1.915	2.141	2.1618
483	758	592	34	82	72	247	5.107	438	257	779	03
2.295	3.768	1.977	1.7218	2.645	1.980	3.755	1.9465	2.961	0.704	2.696	1.2322
042	096	212	05	253	547	891	21	845	21	256	47
1.693	1.769	1.289	1.6011	2.057	3.711	1.476		2.450	1.823	2.259	
961	089	857	36	372	336	691	5.107	837	29	583	5.107
2.579	2.814	2.260	1.5217	0.924	2.569	1.500	1.5126	1.819	1.257	1.876	1.4766
819	422	669	77	725	745	975	82	483	233	204	91
1.659	2.681	1.913		1.474	2.583	1.440	0.6735	3.532	1.695	3.860	1.5009
297	053	22	5.107	531	619	488	83	945	871	049	75
2.382	1.763	2.189	1.1211	0.634	1.298	1.271		2.136	4.096	1.424	1.4404
953	74	582	47	185	928	059	5.107	282	41	362	88
2.783	0.681	3.528	0.4933	1.054	2.279	0.835	1.1797	1.591	3.597	1.373	1.2710
719	86	656	18	286	96	342	96	475	052	674	59
3.702	4.652	2.070	2.4023	1.674	4.397	2.720	1.3145	1.341	2.994	2.161	0.8353
232	499	534	61	931	629	744	67	987	698	803	42
2.398	1.564	1.721		4.087	1.229	1.946	1.5794	3.460	2.332	1.232	
368	775	805	5.107	779	768	521	2	82	72	247	5.107
2.209	1.556	1.601	2.2138	2.274	1.246	2.698	1.5095	2.645	1.980	3.755	1.9465
012	516	136	1	22	049	305	25	253	547	891	21
0.412	4.916	1.521	1.5054	3.236	3.220	1.512	0.9794	2.057	3.711	1.476	
404	563	777	75	14	256	682	32	372	336	691	5.107
0.412	0.790	1.213	0.2556	1.357	4.034	1.493	2.2595	2.579	2.814	2.260	1.5217
014	938	011	81	295	043	023	83	819	422	669	77
0.726	0.177	2.295	1.6565	1.556	1.219	0.213	1.8762	1.659	2.681	1.913	
176	768	66	95	248	095	14	04	297	053	22	5.107
1.997	2.187	1.807	2.3634	2.128	1.724	1.492		2.382	1.763	2.189	1.1211
629	867	191	89	116	023	534	5.107	953	74	582	47
1.191	1.924	2.425	0.8029	1.286	2.961	1.953	1.4243	2.783	0.681	3.528	0.4933
937	113	166	8	818	882	234	62	719	86	656	18
3.692	1.586	4.803	2.1201	4.314	1.466	3.143	1.3736	3.702	4.652	2.070	2.4023
983	588	78	94	494	767	783	74	232	499	534	61
1.684	2.117	1.026		0.364	1.915	2.141	2.1618	2.398	1.564	1.721	
446	991	157	5.107	438	257	779	03	368	775	805	5.107
1.803	3.191	2.654		2.961	0.704	2.696	1.2322	2.209	1.556	1.601	2.2138
541	923	436	5.107	845	21	256	47	012	516	136	1
2.217	1.364	0.255	1.9772	2.450	1.823	2.259		2.865	1.105	1.233	
185	228	681	12	837	29	583	5.107	441	934	389	5.107
3.158	1.882	1.656	1.2898	1.819	1.257	1.876	1.4766	1.357	4.034	1.493	2.2595
38	056	595	57	483	233	204	91	295	043	023	83
2.520	3.238	2.363	2.2606	3.532	1.695	3.860	1.5009	1.556	1.219	0.213	1.8762
815	732	489	69	945	871	049	75	248	095	14	04
2.579	2.814	2.260	1.5217	0.924	2.569	1.500	1.5126	1.819	1.257	1.876	1.4766
819	422	669	77	725	745	975	82	483	233	204	91
1.659	2.681	1.913		1.474	2.583	1.440	0.6735	3.532	1.695	3.860	1.5009
297	053	22	5.107	531	619	488	83	945	871	049	75
2.382	1.763	2.189	1.1211	0.634	1.298	1.271		2.136	4.096	1.424	1.4404
953	74	582	47	185	928	059	5.107	282	41	362	88
2.783	0.681	3.528	0.4933	1.054	2.279	0.835	1.1797	1.591	3.597	1.373	1.2710
719	86	656	18	286	96	342	96	475	052	674	59
3.702	4.652	2.070	2.4023	1.674	4.397	2.720	1.3145	1.341	2.994	2.161	0.8353
232	499	534	61	931	629	744	67	987	698	803	42

2.398368	1.564775	1.721805	5.107	4.087779	1.229768	1.946521	1.57942	3.46082	2.33272	1.232247	5.107
2.209012	1.556516	1.601136	2.21381	2.27422	1.246049	2.698305	1.509525	2.645253	1.980547	3.755891	1.946521
0.412404	4.916563	1.521777	1.505475	3.23614	3.220256	1.512682	0.979432	2.057372	3.711336	1.476691	5.107
0.412014	0.790938	1.213011	0.255681	1.357295	4.034043	1.493023	2.259583	2.579819	2.814422	2.260669	1.521777
0.726176	0.177768	2.29566	1.656595	1.556248	1.219095	0.21314	1.876204	1.659297	2.681053	1.91322	5.107
1.997629	2.187867	1.807191	2.363489	2.128116	1.724023	1.492534	5.107	2.382953	1.76374	2.189582	1.121147
1.191937	1.924113	2.425166	0.80298	1.286818	2.961882	1.953234	1.424362	2.783719	0.68186	3.528656	0.493318
3.692983	1.586588	4.80378	2.120194	4.314494	1.466767	3.143783	1.373674	3.702232	4.652499	2.070534	2.402361
1.684446	2.117991	1.026157	5.107	0.364438	1.915257	2.141779	2.161803	2.398368	1.564775	1.721805	5.107
1.803541	3.191923	2.654436	5.107	2.961845	0.70421	2.696256	1.232247	2.209012	1.556516	1.601136	2.21381
2.217185	1.364228	0.255681	1.977212	2.450837	1.82329	2.259583	5.107	2.865441	1.105934	1.233389	5.107
3.15838	1.882056	1.656595	1.289857	1.819483	1.257233	1.876204	1.476691	1.357295	4.034043	1.493023	2.259583
2.520815	3.238732	2.363489	2.260669	3.532945	1.695871	3.860049	1.500975	1.556248	1.219095	0.21314	1.876204
2.967042	1.897054	0.80298	1.91322	2.136282	4.09641	1.424362	1.440488	2.128116	1.724023	1.492534	5.107
2.298446	0.468165	2.120194	2.189582	1.591475	3.597052	1.373674	1.271059	1.286818	2.961882	1.953234	1.424362
1.459319	0.880196	2.936515	5.107	1.341987	2.994698	2.161803	0.835342	4.314494	1.466767	3.143783	1.373674
1.078483	2.190758	3.635592	2.070534	3.46082	2.33272	1.232247	5.107	0.364438	1.915257	2.141779	2.161803

Table 3.2 shows the simulated samples from Inverse Rayleigh Distribution and HDEWMA control charting statistics calculations.

4. CONCLUSIONS AND RECOMMENDATIONS

In this paper, we have proposed a Hybrid Double Exponentially Weighted Moving Average HDEWMA control chart. We used Monte Carlo Simulations to determine the constants of the control chart for various values. The control chart is proposed for monitoring the inverse Rayleigh lifetimes. The calculated results are for the shifts in scale parameter i.e. 10% - 40% increasing shifts. The result shows that the proposed HDEWMA chart out performs the competitor control charts (i.e. EWMA & CUSUM Charts). The proposed chart performance is assessed for SRS and RSS schemes. It is observed that the HDEWMA using RSS out performs the HDEWMA using SRS, EWMA using RSS, EWMA using SRS, CUSUM using RSS and CUSUM using SRS.

The most efficient results for the proposed chart is obtained for 40% increasing shift. The study can be extended by using median rank set sampling, double rank set sampling and extreme rank set sampling. Since the proposed chart uses the sensitizing rule; single point falls out-of-control so the work can be extended using 2/3, 3/4 out of control sensitizing rule. The similar type of control charts can be developed for other skewed distributions e.g. Rayleigh, Gamma, Inverse Gamma etc.

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