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MONITORING INVERSE RAYLEIGH DISTRIBUTED LIFETIMES USING DEWMA CONTROL CHART

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ABSTRACT
In this paper, we proposed a Double Exponentially Weighted Moving Average DEWMA control chart. The proposed control chart is based on Inverse Rayleigh Distributed lifetimes using simple random sampling. Out-of-control-Average Run Length (ARL_1) is used to evaluate the performance of the proposed control chart. The DEWMA control chart is compared with traditional/simple EWMA and CUSUM control charts. A simulated example is used to compare the proposed DEWMA, traditional/simple EWMA chart control chart. It is observed that the proposed DEWMA control chart performs simple EWMA control charts. The DEWMA control chart can be used for efficient monitoring of the production process in manufacturing industries where the data is coming from inverse Rayleigh Distribution.

Keywords: EWMA, CUSUM, DEWMA, average run length (ARL).

1. INTRODUCTION

Shewhart (1931) introduced the idea of control charting. Control charts are consistently employed by practitioners to examine one or more quality attributes which are directly or indirectly connected with a production process. Shewhart laid the groundwork for the control chart and the concept of statistical control by carefully designed experiments. The control chart is a graph used to show how a process changes over time. Control chart consists of two main categories, variable control chart and attribute control chart. Variable control chart is used to assess changes in a procedure with data that measured on a continuous scale, it is further consists of the $\overline{X}$ Chart, R-Chart, S-Chart, Exponentially Weighted Moving Average (EWMA) Chart, and Cumulative Sum (CUSUM) Chart."Attribute control chart is based on a discrete measurement scale. The average run length is the number of samples taken before an out of control signal is indicated. The control chart with the lowest ARL1 value is one of the best choices for detecting assignable cause. (Montgomery 2009a).

The use of statistical quality control charts in many sectors of life revealed that nearly all of the control charts are meant to offer data on the quality characteristic/studied variable. If we can obtain some information about another variable(s) that is connected with our variable of interest, we may use the efficient charting statistic to improve the control chart's efficiency. The additional information is known as auxiliary information, and the variable that provides it is known as auxiliary variable. This additional/auxiliary information is utilized to estimate the unknown parameters at various times in the survey sampling process (Montgomery (2009a).

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Haq (2013) suggested the New -EWMA (HEWMA) chart for the process mean. The control chart's performance is monitored using the Average Run Length (ARL). The Average Run Length is the average number of points drawn on a control chart before an out of control condition is signaled. Montgomery (2009) considers the control chart with the lowest ARL_1 score to be one of the best options for detecting assignable cause. Rayleigh distribution, one of the well-known continuous distributions created by Lord Rayleigh, was discussed by Hussain and Ahmad (2014). Raza et al. (2015) suggested EWMA and DEWMA control charts for gamma-distributed lifetimes with Type-I censoring.

The concept of conditional anticipated values was employed to keep track of the mean level. The average run length was used as a metric in the performance assessments. Raza et al. (2016) wanted to look at EWMA control charts with Type-I censoring for Poisson-exponential lifetimes. Simple random sampling and rank set sampling, two extensively used sample strategies, were utilized. Monitoring mean level shifts with censored data piqued the interest of numerous applicable challenges.

Alkahtani (2013) investigated the effect of non-normality on the average run length (ARL) performance of EWMA and DEWMA using various skewed (Gamma) and symmetric non-normal (t) distributions. The paper dealt with the modelling of heterogeneity in lifetime processes using the mixture of the inverse Rayleigh distribution, and the focus was on the Bayesian inference of the mixture model using non-informative (Jeffreys and uniform) and informative (gamma) priors. Ali (2015), Abdelkhalek (2020), Alevizakos and Koukouvinos (2020) and Shahid and Rahee (2019) proposed and studied a new flexible extension of the inverse Rayleigh model.

Abbas et al. (2017) explained process monitoring is an essential element for an improved quality of final products. A variety of tools were used for it; control charts were one of these choices. Raza et al. (2018), Abbasi et al. (2018) stated the process structures of manufacturing industry efficiently modeled control charts. Kowsalya et al. (2019) developed a new attribute control chart based on inverse Rayleigh distribution under a time truncated life test. Feng et al. (2020) proposed a test statistic based on the "divide-and-conquer" strategy, and integrated this into the multivariate EWMA charting scheme for Phase II process monitoring.

The main ideology was to calculate and combine the T_2 statistics on low-dimensional sub-vectors together. Essentially the proposed procedure was distribution-free and computation efficient. Raza et al. (2020) proposed DEWMA chart to monitor the mean of Weibull distribution in the presence of type-I censored data. Their focus was to use the conditional median (CM) for the imputation of censored observations. The performance of the control chart was determined by the average run length (ARL). They compared CM-DEWMA control chart with CM-based DEWMA control chart.

In this paper we propose a Double Exponentially Weighted Moving Average DEWMA control chart by inverse Rayleigh distribution. The performance of the proposed chart will be determined using out-of-control-Average Run Length (ARL_1).

2. METHODOLOGY

2.1 The Proposed Double EWMA Control Chart

For monitoring the data is generated from Inverse Rayleigh Distribution. We define a DEWMA control chart. The PDF of the inverse Rayleigh distribution with scale parameter θ is:

$$f(x) = \left(\frac{2\theta}{x^3}\right)\exp\left(-\frac{\theta}{x^2}\right) \qquad x,\theta > 0 \tag{1}$$

The corresponding cumulative distribution function is,

$$F(x) = \exp\left(-\frac{\theta}{x^2}\right) \qquad x,\theta > 0 \tag{2}$$

Let $X_1, X_2, \ldots,$ be a sequence of IID random variables from the Inverse Rayleigh Distribution.

EWMA statistics is calculated as:

$$E_t = \lambda_1 \overline{X}_i + (1-\lambda_1) E_{t-1} \tag{3}$$

Now, the plotting statistic of DEWMA is calculated as: $HE_t = \lambda_2 E_t + (1-\lambda_2) HE_{t-1}$ (4)

Compute the DEWMA statistics for different subgroup size m and sample size n. Take the $(1-p)-\text{th}$ quantile point of the charting statistics values to define the UCL of the chart. Where, p is specified as 0.0027 (Montgomery et al. 1994).
The control limits of the proposed chart will be derived as:
$E(HE_t) = \mu$
So control limits are:
$UCL/LCL = \mu \pm L_i\sqrt{Var(HE_t)}$
Where L_1 and L_2 shows the 99.73 quantile point of the distribution under study.
Algorithm 1:

1. Initiate the algorithm by fixing the sample size, n and group size m. Estimate the parameter using phase-I data if it is unknown.
2. The sample is selected using SRS and RSS schemes.
3. Calculate the charting statistics of proposed DEWMA statistics for given sample size "n". Now take $(1-p)th$ quantile point, here α is predefined,
4. Repeat Step No.2 *1000 number of time*& compute the average, it will out-turn the Upper Control Limit.
5. Hatch the values of DEWMA monitoring statistic with respect to the subgroup numbers. Record the sample number at which the monitoring statistic falls outside the control limit,
6. Repeat Step 4 a large number of times and calculate the mean of the subgroup numbers at which the process first time crossed the UCL. This will result in ARL_0 if the subgroups are generated from the in-control process. For the out-of-control ARL computation, introduce a shift in the data and test it against the control limit constructed using in-control data. Repeat steps 4-5 and calculate the average of subgroups falling outside the UCL, which is ARL_1.

3. RESULTS AND DISCUSSION

To study the features of the proposed DEWMA control chart, we have simulated the data and applied the DEWMA chart. The procedure proposed in section 2 is evaluated using simple random sampling. The EWMA weight (λ_1) and DEWMA weight (λ_2) are fixed for evaluation purpose. The simulated distribution is found to follow Inverse Rayleigh distribution. Therefore, the limits are based on Inverse Rayleigh distribution. Among these, some results are presented here for discussion purposes. Table1 gives the ARLs of the proposed DEWMA control chart.

Table 1. ARL_1 and SDRL values for DEWMA and EWMA control charts for n=3,5 with specified location parameter θ=1.5 using ARL_0=200, λ_1=λ_2=0.1

Shifts in location parameter		DEWMA Chart ARL₁	DEWMA Chart SDRL	EWMA Chart ARL₁	EWMA Chart SDRL
		n=3			
5%	**Increasing**	160.87	158.04	185.70	163.75
10%		152.76	79.67	158.71	150.22
15%		63.15	56.65	67.42	60.22
20%		11.98	19.16	31.31	12.92
25%		5.39	9.08	9.97	6.94
30%		5.71	3.34	5.34	5.12
5%	**Decreasing**	180.42	157.10	189.66	176.42
10%		153.86	124.93	167.73	154.88
15%		75.81	53.07	108.56	73.13
20%		36.76	16.42	18.40	33.87
25%		5.47	7.07	10.66	5.23
30%		2.08	3.32	2.82	2.33
		n=5			

Shifts in location parameter		DEWMA Chart ARL$_1$	DEWMA Chart SDRL	EWMA Chart ARL$_1$	EWMA Chart SDRL
5%	**Increasing**	125.75	115.64	150.36	124.86
10%		86.99	57.75	107.74	86.53
15%		30.29	18.69	31.58	29.91
20%		6.15	7.61	10.95	5.46
25%		1.74	3.49	3.56	1.74
30%		1.00	1.50	1.76	1.22
5%	**Decreasing**	185.08	100.15	167.27	184.74
10%		95.09	86.98	148.60	94.60
15%		45.07	19.61	33.16	44.15
20%		4.75	6.31	8.83	4.02
25%		2.63	3.22	4.28	2.43
30%		1.52	1.08	1.55	1.25

Table 1 represents the out-of-control values of ARL for DEWMAandEWMA control charts of sample size n=3. The samples are generated from Inverse Rayleigh Distribution with parameter θ=1.5. The in-control ARL (ARL_0) is fixed at 200;we have simulated results 10,000 times using Monte Carlo Simulations (MC Simulation). It is noticed that DEWMA chart outperforms the EWMA control chart for all the shift sizes.

Figure 1. ARL_1 for DEWMAand EWMA with θ=1.5, ARL_0=200.

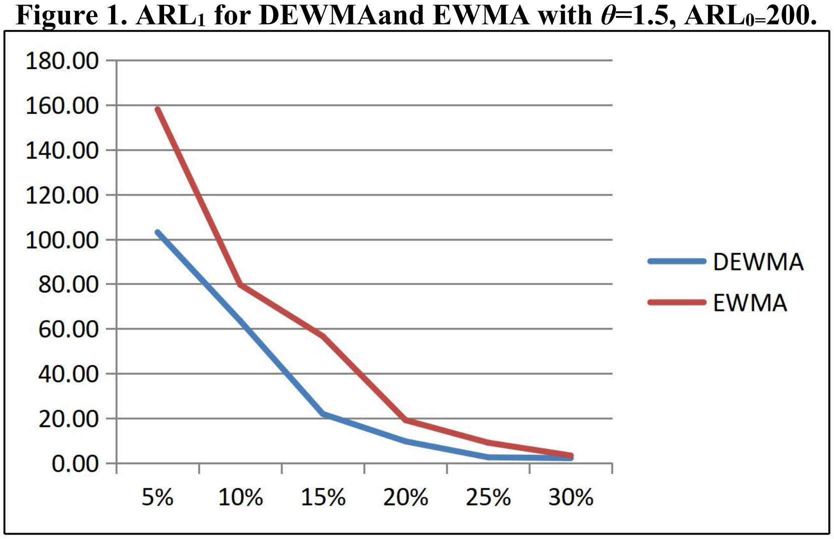

Figure 1 graphically represents the out-of-control values of ARL for DEWMAand EWMA control charts of sample size n=3. The samples are generated from Inverse Rayleigh Distribution with parameter θ=1.5. The in control ARL (ARL_0) is fixed at 200, We have simulated results 10,000 times using MC Simulations. It is noticed thatARL_1 values for DEWMA control charts are less than the EWMA charts ARL_1 values. The DEWMA chart out performs EWMA charts for different shift sizes.

Table 2 Simulated Samples and Calculation of statistics of DEWMA chart

Inverse Rayleigh R.No			DEWMA Statistics	Inverse Rayleigh R.No			DEWMA Statistics	Inverse Rayleigh R.No			DEWMA Statistics
2.283293	0.245293	3.243403	0.856573	2.225195	2.055401	0.330907	0.773359	2.510258	1.246788	1.623823	0.461071
1.984712	1.365452	1.834227	0.68692	0.534097	2.826074	1.601667	0.642021	2.482996	2.556775	1.491422	1.863349
1.620366	0.960469	0.783701	0.473115	1.073831	0.069769	4.249208	0.62307	2.153289	1.750401	0.634002	1.475744
0.560821	0.179725	3.361792	0.869618	1.51133	2.571372	1.257726	1.089032	1.628636	0.113203	2.013523	2.053113
1.08805	2.297599	1.883505	1.574212	3.310079	0.062917	0.701192	1.007467	1.337106	0.73471	2.775078	4.72037
0.421724	1.385212	2.555143	4.746741	1.93202	1.104528	1.173459	4.843329	1.037398	2.108687	3.058501	1.447277

0.232555	0.444333	1.052047	0.378492	1.957046	1.375485	3.518446	2.012949	1.919797	3.122519	1.140783	1.250895
0.367077	0.996002	1.380376	4.572607	2.820673	1.062792	0.878194	4.628388	1.555817	1.349676	0.52613	0.758843
0.431362	0.533662	0.676884	0.234462	0.638433	3.613238	0.996613	1.555272	2.156214	2.444199	2.044999	1.352637
0.183477	0.372186	1.452529	1.481094	0.829273	0.386224	0.070263	1.375648	1.294619	1.923996	1.664529	4.976406
1.094484	2.100983	1.060564	1.926569	1.340589	1.362131	1.204484	4.771426	1.751515	1.062724	2.102256	1.053687
0.763639	1.888091	2.30836	0.2612	1.095448	2.063523	1.780124	1.2505	2.562698	0.049882	2.662868	0.102264
2.988125	1.05877	4.62587	1.979913	4.14922	0.702009	2.758892	0.727014	2.782672	4.321078	1.894444	1.78004
1.349809	1.524204	0.319122	4.366764	0.141991	1.025818	1.934415	1.234314	1.87675	1.33897	1.13956	4.229386
1.239719	2.439865	1.814312	4.219791	2.922766	0.57729	2.427435	1.065262	1.338317	1.241381	0.944797	2.085482
1.629191	0.366008	0.052391	1.140392	1.581235	0.867781	1.552733	5.092701	2.042451	0.961112	0.248187	4.155886
2.344012	1.487305	1.512951	1.19591	1.05571	0.423996	0.973942	1.192677	0.47854	3.571653	0.764041	2.248972
2.489447	2.395356	1.580455	1.554862	3.080143	1.244695	3.361868	0.994152	0.775904	0.903001	0.049052	0.978845
2.578596	1.189878	0.457745	1.421394	2.134961	3.319286	1.412727	0.846845	1.416694	1.684411	0.6487	4.854947
2.281097	0.040092	2.039061	1.800086	0.631622	2.635936	1.296791	0.974114	1.178698	2.548098	1.315619	0.931328
1.326651	0.414525	2.462465	4.797552	0.675203	2.373137	2.141129	0.312462	3.573489	0.674367	2.77582	0.485313
1.032688	1.926163	3.567178	1.356892	2.583211	1.701452	1.1779	4.81959	0.304727	1.135152	1.181386	1.920533
1.869511	3.225749	1.61647	0.762321	1.812505	1.566259	3.18281	1.836841	2.818381	0.475879	2.420415	0.643853
0.84807	1.460971	0.909581	0.703462	1.309081	3.540448	0.6019	4.165644	1.494301	1.694946	1.90267	4.548948
1.671224	2.309687	1.267048	1.172297	0.615054	2.544848	1.359007	1.323412	1.353877	0.514288	1.117982	0.952529
1.419303	2.118529	1.634806	5.07444	1.29136	2.047943	0.833753	0.335957	2.759755	1.239638	3.611023	1.370347
2.034593	0.970716	1.495655	0.570123	0.132391	0.740292	1.016322	4.16505	1.39961	4.039325	0.603882	0.895298
2.397245	0.660998	3.267078	0.38793	0.220499	1.320081	0.356473	0.807473	0.749787	3.070521	0.497719	0.841542
2.726457	4.608987	1.952427	2.291584	0.843336	4.12664	2.29341	0.938411	0.64952	2.426793	1.300138	0.261463
2.060404	0.900615	0.978167	4.664184	4.051809	0.913608	1.617305	0.68056	2.606493	2.003012	0.463431	5.0751
1.970113	1.101506	0.621557	1.732305	2.015831	1.080506	2.547032	0.77974	1.799007	1.567985	3.035286	1.412295
0.241027	4.858908	0.953206	0.687937	3.026631	2.615071	1.354162	0.823213	1.299027	3.177545	0.800556	5.082722
0.31937	0.034154	1.059281	0.12037	1.020252	3.755225	0.624491	2.098855	2.044759	1.879497	1.778193	0.708207
0.063533	0.023552	2.214682	0.813085	0.689131	0.793831	0.742029	1.429982	0.934339	2.150306	1.066844	4.887749
1.151333	1.366976	0.974141	2.301457	1.601791	1.000059	0.783546	4.460355	2.11616	1.751651	1.385094	0.845442
0.714777	1.536538	1.677921	0.197144	0.458313	2.896476	1.885013	0.769673	1.784315	0.123485	3.478077	0.234777
3.302915	1.31364	4.264264	1.688844	3.544149	1.368724	2.486912	0.587671	3.414005	4.37474	1.990134	2.371865
1.253768	1.465628	0.459569	4.626433	0.104744	1.366173	2.11876	1.437529	1.864755	0.842774	0.801253	5.046185
1.478807	2.42147	2.608256	4.793887	2.560966	0.061752	2.47747	0.495862	1.723544	0.70073	1.338455	2.153811
1.451392	1.337544	0.476371	1.301077	1.58228	1.064598	1.3131	4.619207	2.685822	1.064972	1.063715	4.891021
2.580902	1.747503	1.546061	0.692266	1.647434	0.414648	1.009474	0.5815	0.610922	3.117064	0.974813	2.102495
2.03479	2.977211	2.025403	1.63703	3.501715	1.257685	2.932386	0.771526	1.30945	0.598468	0.125826	1.286109
2.783241	1.20553	0.065679	1.410007	1.469423	3.642116	0.991367	1.343767	1.57882	1.497823	0.898086	5.007658
1.863769	0.22545	1.750995	2.056745	1.084457	3.417214	0.668121	0.864252	1.245569	2.061225	1.053152	0.527829
1.064208	0.562687	2.847706	4.364936	0.495679	2.226154	2.016969	0.512904	3.66354	0.904503	2.379448	0.826373
0.775267	1.762357	2.736348	2.027687	3.209124	1.76902	0.977813	5.039347	0.43663	1.745224	1.39204	1.27239
1.899478	3.638269	1.819428	1.678009	1.777084	1.365722	3.471611	1.892475	2.280893	0.258568	2.55882	0.393701
0.782307	1.586709	0.374675	1.421315	1.065636	3.269202	0.693846	4.226197	1.746675	0.873233	2.083046	4.513779
2.240207	2.166095	2.249363	0.688547	0.043489	2.440947	0.907192	0.731345	0.932038	0.650358	1.598664	1.159941
0.885387	2.028269	1.710881	4.477443	0.987229	1.916508	1.155717	0.067471	3.351298	1.61786	3.483624	0.67706
2.356073	1.015345	2.114452	0.39792	0.09999	0.372431	1.227032	4.259708	1.555962	3.97523	1.080437	0.896881
2.42371	0.169666	3.306279	0.255669	0.601103	1.545721	0.822321	0.357914	0.770159	2.943622	0.499634	0.340424
2.867166	4.584461	1.20761	1.527331	1.175591	4.147705	1.951334	0.423635	1.141212	2.772022	1.757194	0.697571
2.11563	1.547266	1.354942	4.25598	3.93777	0.299188	1.095541	1.344374	3.275086	1.648269	0.974769	4.35944
1.386928	1.438483	0.732599	1.270544	1.876774	0.540149	2.084125	0.711088	1.979382	1.326729	3.699979	1.925551
0.487751	4.151407	1.233951	0.728603	2.893778	2.397916	0.827189	0.020501	1.503598	2.90779	1.208811	4.309853
0.273096	1.220343	4.973386	0.699092	1.869396	2.909009	0.444392	1.75259	0.156094	1.925926	1.339782	1.353324
2.179687	1.38966	1.327749	1.15653	1.645514	0.383274	0.926226	1.382803	1.148934	3.858657	1.001035	1.30204
2.364	2.277638	1.968317	1.320847	2.594826	0.928909	3.137408	0.659853	1.061022	0.897403	0.405103	1.671243
2.749539	1.84226	0.002473	1.790666	1.374707	3.297337	0.647773	0.826311	2.015483	0.831361	1.21294	4.993628

1.316427	0.174114	1.631199	1.430595	0.951946	3.085044	0.447855	1.021075	0.982049	2.247921	1.011938	1.177777
0.693426	0.770254	1.949092	4.648458	0.597999	2.083782	1.472969	0.761269	4.004781	1.413524	3.051089	0.54946
1.071705	1.237066	3.097216	1.698019	3.137791	1.491681	0.318876	4.981474	0.371878	1.506836	1.842462	1.259031
1.500142	3.763073	1.783912	1.36773	1.646689	1.153764	3.300216	1.665647	2.758292	0.105816	1.766519	0.593606
1.185904	1.028705	0.946425	0.930581	1.92729	3.169098	1.289293	4.178344	2.301063	1.766458	2.241179	4.511025
2.208902	2.4772	1.522004	0.967976	0.742381	2.424165	1.031588	1.28749	1.620913	0.451325	1.291263	0.536321
0.978695	2.217237	0.950699	4.546897	1.445413	1.789768	0.488734	0.450964	2.829484	1.467557	3.574268	0.582047
1.54829	1.073611	2.092547	0.625396	0.425007	0.449332	1.049347	4.270427	1.935009	4.027693	0.530736	0.70489
2.309793	0.094923	3.079306	0.13004	0.981407	1.570068	0.66256	0.611706	1.36381	3.200256	0.531942	0.991268
2.986761	4.641214	1.887477	2.385372	1.0128	3.747735	2.416588	0.801263	0.855013	2.441672	1.762736	0.203554
1.766323	0.80118	1.529139	4.27992	3.136597	0.958453	1.388683	1.096297	2.608638	2.1807	1.104202	4.844629
2.02819	0.580779	0.888671	1.819988	1.486208	1.136755	1.910064	1.115611	1.820979	1.227916	3.696327	1.323019
0.45028	4.857351	1.520528	1.209021	2.793771	2.325567	0.974076	0.113152	1.395422	3.601395	0.84719	4.784595
0.340973	0.753999	0.943686	0.038627	1.18258	3.834507	1.240659	1.62504	2.106262	2.063344	1.356828	1.214201
0.500895	0.581154	2.172791	1.251904	1.406061	0.650304	0.706822	1.240852	1.171647	2.322479	1.616405	4.712631
1.253422	1.268459	1.150743	1.466162	1.949521	1.066086	1.147779	4.536585	2.248109	0.93379	1.77867	0.987387
1.046998	1.11834	1.5208	0.045899	0.52458	2.602696	1.000918	0.980089	2.293446	0.05595	3.468813	0.421131
3.177299	0.651703	4.350139	1.881217	3.372117	0.523339	2.365231	0.628325	3.582937	3.876085	1.20456	2.261386
0.96088	1.421394	0.530274	4.149446	0.020078	1.45112	2.081821	1.183485	1.574515	1.381448	1.489125	4.11769
1.563294	3.007827	2.06192	4.614156	2.806509	0.391616	1.705779	0.804878	1.212633	0.704458	1.288747	1.796323
1.808499	0.824717	0.707889	1.698579	1.698271	1.237667	1.434207	4.641376	2.174157	0.609736	0.377951	4.578443
2.188773	1.613236	0.835417	0.791294	1.270703	0.999483	1.765974	1.327396	0.403049	3.78159	1.280466	1.408542
1.950248	2.989123	1.9859	1.921372	3.41219	0.704652	3.358534	0.723413	0.733552	1.130009	0.666398	1.563314
1.861221	2.162565	1.837391	0.720702	0.882037	2.343799	0.943124	1.079135	1.441977	0.293615	1.675869	0.598302
0.887939	2.039787	1.144845	4.612109	1.164881	2.377369	0.829652	0.208131	3.035445	1.358317	3.552224	0.875715
1.908426	0.77283	2.041996	0.750341	0.529688	1.082284	0.867308	4.525097	1.379954	3.740418	0.609654	0.949
2.554188	0.301991	3.043825	0.417828	0.701922	2.212082	0.806659	1.135574	0.594481	2.902668	0.82844	0.479639
3.498007	4.369935	1.907532	1.530345	1.281799	3.858342	2.533592	1.155934	0.557999	2.224123	1.377079	0.544375
1.523149	0.588321	1.153726	4.99419	3.253551	0.564818	1.828488	0.605076	2.905389	2.030048	0.769062	4.190894
1.776109	1.348569	1.144544	1.797282	1.886571	0.610545	2.507233	0.940272	1.849373	1.392162	3.21256	1.727943
0.440511	4.708641	0.68501	1.005729	2.682942	2.594569	1.141421	0.020181	1.67622	2.874974	1.099851	4.128034
0.26507	0.268569	0.552818	0.184608	1.319962	3.997473	0.863525	1.475144	1.670952	1.967633	1.447718	1.378152
0.034046	0.325932	1.495062	1.040608	1.257238	1.05121	0.033317	1.350584	1.620415	1.858423	1.755452	4.653985
1.034861	2.177328	1.085706	2.26643	1.584095	1.212857	1.347488	4.734915	1.832075	1.445598	2.186236	0.467033
0.872445	1.667294	1.430878	0.150226	0.94032	2.479603	0.955663	1.235924	2.457367	0.53051	3.440352	0.325696
3.650339	1.525661	4.678535	1.941	3.47299	0.611682	2.597637	0.902626	3.0783	3.710147	1.653775	2.205607
1.612224	1.856169	0.801649	4.998692	0.225837	1.527904	1.757575	1.258634	1.95106	1.052681	0.816982	4.243941
1.459568	2.614983	2.026622	4.330803	2.473656	0.017493	2.335558	1.223669	1.735443	1.073702	0.79518	1.99011
1.598042	0.971226	0.639856	1.895342	1.790039	1.705164	1.765389	4.310651	2.537648	0.760859	0.47094	5.048343
2.60546	1.603081	0.952932	1.065822	1.74103	0.470639	1.570123	0.508298	1.30962	3.066941	0.89965	1.635878
2.28755	2.374173	1.398692	2.08707	2.673314	1.627047	3.322755	1.197412	0.946408	1.106681	0.005345	1.157044
2.859665	1.836173	0.509988	1.658641	1.209513	3.228059	0.965213	0.737772	1.912508	0.939605	1.066288	4.454951
1.865935	0.355204	2.108663	1.28513	1.030903	2.728198	1.108872	0.989726	0.567554	2.689458	1.010611	0.90087
1.039013	0.20496	2.633649	4.402941	0.38308	2.07385	1.615852	0.403262	3.348881	1.388715	2.177248	1.303414
1.500142	3.763073	1.783912	1.36773	1.646689	1.153764	3.300216	1.665647	2.758292	0.105816	1.766519	0.593606
1.185904	1.028705	0.946425	0.930581	1.92729	3.169098	1.289293	4.178344	2.301063	1.766458	2.241179	4.511025
2.208902	2.4772	1.522004	0.967976	0.742381	2.424165	1.031588	1.28749	1.620913	0.451325	1.291263	0.536321
0.978695	2.217237	0.950699	4.546897	1.445413	1.789768	0.488734	0.450964	2.829484	1.467557	3.574268	0.582047
1.54829	1.073611	2.092547	0.625396	0.425007	0.449332	1.049347	4.270427	1.935009	4.027693	0.530736	0.70489
2.309793	0.094923	3.079306	0.13004	0.981407	1.570068	0.66256	0.611706	1.36381	3.200256	0.531942	0.991268
2.986761	4.641214	1.887477	2.385372	1.0128	3.747735	2.416588	0.801263	0.855013	2.441672	1.762736	0.203554
1.766323	0.80118	1.529139	4.27992	3.136597	0.958453	1.388683	1.096297	2.608638	2.1807	1.104202	4.844629
2.02819	0.580779	0.888671	1.819988	1.486208	1.136755	1.910064	1.115611	1.820979	1.227916	3.696327	1.323019
0.45028	4.857351	1.520528	1.209021	2.793771	2.325567	0.974076	0.113152	1.395422	3.601395	0.84719	4.784595

0.340973	0.753999	0.943686	0.038627	1.18258	3.834507	1.240659	1.62504	2.106262	2.063344	1.356828	1.214201
0.500895	0.581154	2.172791	1.251904	1.406061	0.650304	0.706822	1.240852	1.171647	2.322479	1.616405	4.712631
1.253422	1.268459	1.150743	1.466162	1.949521	1.066086	1.147779	4.536585	2.248109	0.93379	1.77867	0.987387
1.046998	1.11834	1.5208	0.045899	0.52458	2.602696	1.000918	0.980089	2.293446	0.05595	3.468813	0.421131
3.177299	0.651703	4.350139	1.881217	3.372117	0.523339	2.365231	0.628325	3.582937	3.876085	1.20456	2.261386
0.96088	1.421394	0.530274	4.149446	0.020078	1.45112	2.081821	1.183485	1.574515	1.381448	1.489125	4.11769
1.563294	3.007827	2.06192	4.614156	2.806509	0.391616	1.705779	0.804878	1.212633	0.704458	1.288747	1.796323
1.808499	0.824717	0.707889	1.698579	1.698271	1.237667	1.434207	4.641376	2.174157	0.609736	0.377951	4.578443
2.188773	1.613236	0.835417	0.791294	1.270703	0.999483	1.765974	1.327396	0.403049	3.78159	1.280466	1.408542
1.950248	2.989123	1.9859	1.921372	3.41219	0.704652	3.358534	0.723413	0.733552	1.130009	0.666398	1.563314
1.861221	2.162565	1.837391	0.720702	0.882037	2.343799	0.943124	1.079135	1.441977	0.293615	1.675869	0.598302
0.887939	2.039787	1.144845	4.612109	1.164881	2.377369	0.829652	0.208131	3.035445	1.358317	3.552224	0.875715
1.908426	0.77283	2.041996	0.750341	0.529688	1.082284	0.867308	4.525097	1.379954	3.740418	0.609654	0.949
2.554188	0.301991	3.043825	0.417828	0.701922	2.212082	0.806659	1.135574	0.594481	2.902668	0.82844	0.479639
3.498007	4.369935	1.907532	1.530345	1.281799	3.858342	2.533592	1.155934	0.557999	2.224123	1.377079	0.544375
1.523149	0.588321	1.153726	4.99419	3.253551	0.564818	1.828488	0.605076	2.905389	2.030048	0.769062	4.190894
1.776109	1.348569	1.144544	1.797282	1.886571	0.610545	2.507233	0.940272	1.849373	1.392162	3.21256	1.727943
0.440511	4.708641	0.68501	1.005729	2.682942	2.594569	1.141421	0.020181	1.67622	2.874974	1.099851	4.128034
0.26507	0.268569	0.552818	0.184608	1.319962	3.997473	0.863525	1.475144	1.670952	1.967633	1.447718	1.378152
0.034046	0.325932	1.495062	1.040608	1.257238	1.05121	0.033317	1.350584	1.620415	1.858423	1.755452	4.653985
1.034861	2.177328	1.085706	2.26643	1.584095	1.212857	1.347488	4.734915	1.832075	1.445598	2.186236	0.467033
0.872445	1.667294	1.430878	0.150226	0.94032	2.479603	0.955663	1.235924	2.457367	0.53051	3.440352	0.325696
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1.612224	1.856169	0.801649	4.998692	0.225837	1.527904	1.757575	1.258634	1.95106	1.052681	0.816982	4.243941
1.459568	2.614983	2.026622	4.330803	2.473656	0.017493	2.335558	1.223669	1.735443	1.073702	0.79518	1.99011

Table 2 shows the simulated samples from Inverse Rayleigh Distribution and DEWMA control charting statistics calculations.

4. CONCLUSIONS AND RECOMMENDATIONS

In this paper, we have proposed a Double Exponentially Weighted Moving Average DEWMA control chart. We used Monte Carlo Simulations to determine the constants of the control chart for various values. The control chart is proposed for monitoring the inverse Rayleigh lifetimes. The calculated results are for the shifts in scale parameter i.e., 10% - 30% increasing shifts. The result shows that the proposed DEWMA chart outperforms the competitor EWMA control charts. It is observed that the DEWMA chart outperforms the EWMAcontrol chart.The most efficient result for the proposed chart is obtained for 40% increasing shift. The study can be extended by using median rank set sampling, double rank set sampling and extreme rank set sampling.

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